

## THE CHANNEL WAVEGUIDE TRANSFORMER : AN EASILY FABRICATED H-PLANE TRANSITION FOR THE RECTANGULAR TE-10 MODE

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## ABSTRACT

The authors describe an easily fabricated H-plane transformer for use in rectangular waveguide carrying the dominant mode. An approximate theoretical analysis of the structure is presented, and computed results are compared with measurements on transformers at X-band. Design curves are given for transitions from full to one-half and one-quarter height waveguide. The new transformers have been found particularly useful for millimeter-wave mixers and multipliers with split-block construction. The structure can also be used as a transition from rectangular to channel waveguide.

## INTRODUCTION

Stepped or tapered waveguide transformers are essential components in many microwave circuits. At millimeter wavelengths they are especially difficult to fabricate, often requiring copper electroforming with the production of a separate mandrel for each finished piece. We describe a new type of H-plane transformer which can be made quickly and easily using a slitting saw or single point cutting tool. The transformer has already been used successfully in solid state frequency doublers up to 220 GHz and mixers at 115 GHz.

## TRANSFORMER DESCRIPTION AND MEASURED PERFORMANCE

An exploded view of the channel waveguide transformer is shown in Fig. 1d. We have chosen to use the term "channel" waveguide after a similar structure described by Vilmer and Ishii [1], as it contrasts well with the more familiar ridged guide. One can think of the ridge as having been inverted to form a channel along the axis of propagation.

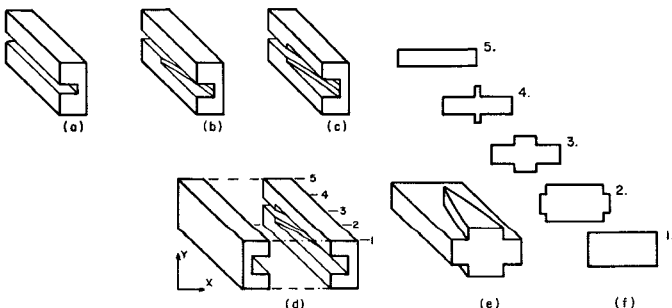
The transformer is most easily fabricated as a split-block structure in which the two halves are joined along a plane of zero transverse current (Fig. 1d). A slitting saw or single point tool is used to cut the reduced height waveguide completely along the two blocks (Fig. 1a). The full height waveguide and transition region are then formed by moving the saw to

each side of the centerline, producing a sloping channel in part of the block (Figs. 1b and 1c). The final result is a length of full height waveguide with sections of its narrow walls tapering in a circular arc towards the center until only the desired reduced height waveguide remains (Fig. 1e). Fig. 1f shows a series of cross sections along the longitudinal axis of the transformer corresponding to the numbered positions in Fig. 1d. The length of the taper is determined by the radius of the slitting saw and the depth of cut, i.e. the waveguide half-width.

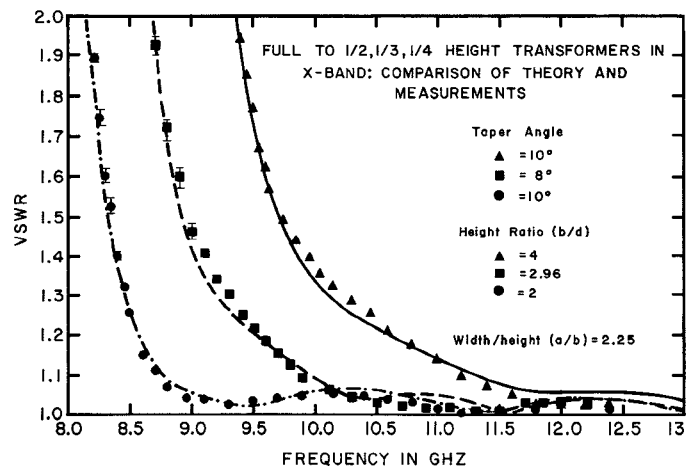
A taper with a linear (rather than circular-arc shaped) profile can be formed by tilting the workpiece and moving it longitudinally under the slitting saw while the transition is being machined.

The measured performance of three X-band channel waveguide transformers with linear tapers is shown in Fig. 2, where VSWR is plotted versus frequency for transitions having input to output height ratios of 2, 3, and 4. Each transformer is approximately one guide wavelength long, as measured in standard X-band waveguide, at 8 GHz.

When the input to output waveguide height ratio is large, this version of the transformer is not well matched at the low frequency end of the waveguide band. As will be shown in the next section, this is due to an increase in the TE-10 mode cutoff frequency inside the



1. (a) View of the right half of the transformer after machining the reduced height waveguide, (b) after the slitting saw has been used to produce one side of the transition from full to reduced height, and (c), the complete right half. (d) An exploded view of the finished transformer. (e) A solid view of the transition region beginning about midway down the length of the taper. (f) A series of cross sections taken along the length of the transformer. The numbers correspond to the positions indicated in (d).



2. Comparison of measured and computed VSWR for three X-band transformers with linear tapers. The lines are the computed values; points, the measured values. Error bars reflect the measurement uncertainties.

transition region. The problem is readily overcome by adding one step to the fabrication process. Upon completing the reduced height waveguide section, the slitting saw is moved to the center of the transition region and plunged into the narrow wall, producing in it, a circular-arc shaped bulge. Electrically, the overall effect is to reduce the value of the TE-10 mode cutoff frequency in the transformer. The substantial improvement in the low frequency performance can be

seen in Fig. 3. The plot shows the VSWR of the full to one quarter height transformer of Fig. 2, when a bulge is added to the width of the reduced height waveguide. In this instance the bulge extends the full length of the transition region and increases the reduced height waveguide width by 37% at the midsection of the transformer. Using the bulgy design, the VSWR can be reduced to less than 1.2 over the full waveguide band (see Figs. 5 and 6).

### THEORY

An approximate analysis of a taper of arbitrary cross section between two uniform waveguides propagating a single mode has been given by Johnson [2]. The tapered region is replaced by a series of short butt-jointed uniform waveguides each having its own propagation constant and guide impedance. Letting the number of sections become large and neglecting higher order modes and multiple reflections, Johnson arrived at the following expression for the reflection coefficient of the dominant mode:

$$\Gamma|_{z=0} = \int_0^L \frac{1}{2} \frac{d}{dz} (\ln(Z_c)) \exp \left[ \int_0^z \gamma(z') dz' \right] dz, \quad (1)$$

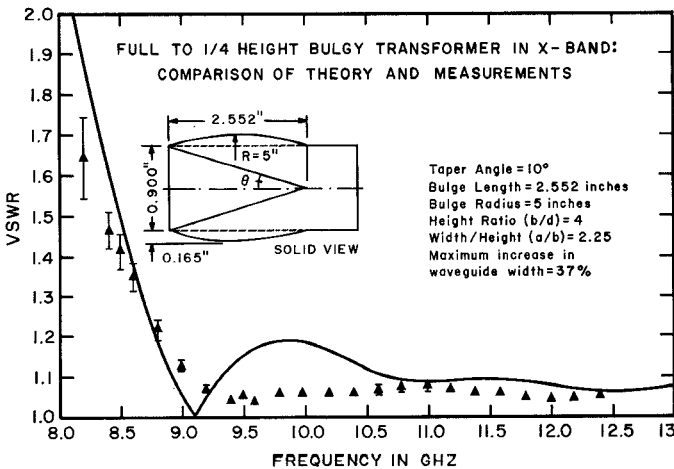
where the integration is over the length,  $L$ , of the transformer. For a gradual transition,  $Z_c(z)$  can be equated with the characteristic impedance of a uniform waveguide having the same cross sectional dimensions as the transformer at position  $z$ .  $\gamma = \alpha(z) + j\beta(z)$  is the propagation constant of the mode in each short section of guide and reduces simply to  $j\beta(z)$  for a lossless transition.  $\beta(z)$  is related to the cutoff wavenumber,  $k_c(z)$  via:  $\beta(z) = [\omega^2 \mu \epsilon - k_c^2]^{1/2}$ , where  $\omega = 2\pi f$  is the radian frequency of the incident wave, and  $\mu$  and  $\epsilon$  are the permeability and permittivity of the medium in the transition. Considering each cross section in Fig. 1f to be that of a uniform waveguide, the value of the cutoff wavenumber, and hence the propagation phase

constant, can be determined using the method of transverse resonance [3]. Approximate expressions for the transverse fields in the cross section can then be used to derive a waveguide characteristic impedance [4,5]. Once the values of  $k_c(z)$  and  $Z_c(z)$  have been determined eq. (1) can be integrated numerically to find  $\Gamma$  at a particular frequency.

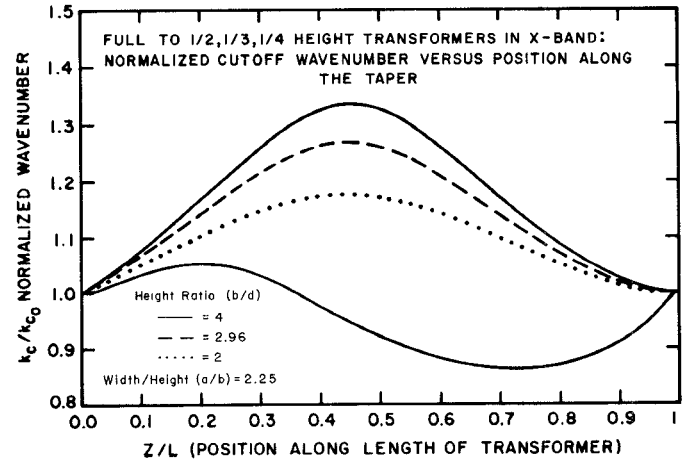
The method of transverse resonance, which is used to determine the cutoff wavenumbers along the length of

the transformer, gives results which agree well with those obtained by a complete field analysis of the channel waveguide structure [6,7]. However, the definition of characteristic impedance in a waveguide is not unique. Schelkunoff [8] discusses the application of the concept of characteristic impedance to waveguide structures. For certain special cases, such as rectangular waveguide, there are three equally useful definitions which differ only by constant factors. However, for non-TEM waveguides generally, and specifically for the channel waveguide structure, there is no obvious choice for the characteristic impedance. The equivalent circuit of a junction between two waveguides with different cross sections can be described by a transformer, which couples between the wave impedances of the propagating mode in the two guides, and a shunt susceptance. To analyze the tapered channel waveguide rigorously by the characteristic impedance method it is necessary to find a definition of characteristic impedance which results in a unity transformer ratio at each incremental change of cross section. Cohn [4] used a particular definition of characteristic impedance in analyzing ridged waveguide, i.e. the voltage from top to bottom across the center of the guide divided by the total longitudinal current along the upper walls. We have found that an analogous definition for channel waveguide gives acceptable agreement with experiment. However, we know of no way to prove that this definition actually does result in a unity transformer ratio.

It is possible to avoid using the concept of a characteristic impedance in analyzing the channel waveguide transformer by employing the more general theory of coupled mode equations given by Solymar [9]. We found however, that it was not practical to predict the transformer performance accurately by this method. The reason appears to be the slow convergence of the series representing the fields [6,7] in the channel waveguide. This problem might be overcome if a numerical finite difference scheme were used to determine the fields and then the small coupling theory of Solymar [9] were



3. Measured and predicted VSWR versus frequency for the full to 1/4 height transformer of Fig. 2 with the addition of a bulge in the reduced height waveguide extending the full length of the taper. The taper half angle  $\theta$  of the linear transition is 10 degrees yielding a transformer length of 6.482 cm. Error bars reflect measurement uncertainties.



4. Predicted values of the normalized cutoff wavenumber versus position for the transformers of Figs. 2 and 3. The lowest curve represents the bulgy full to one-quarter height transformer of Fig. 3. The wavenumber of the channel waveguide,  $k_c$ , is normalized to that of standard X-band waveguide  $k_{c0} = 2\pi/4a$ , where  $a$  is the waveguide half-width.

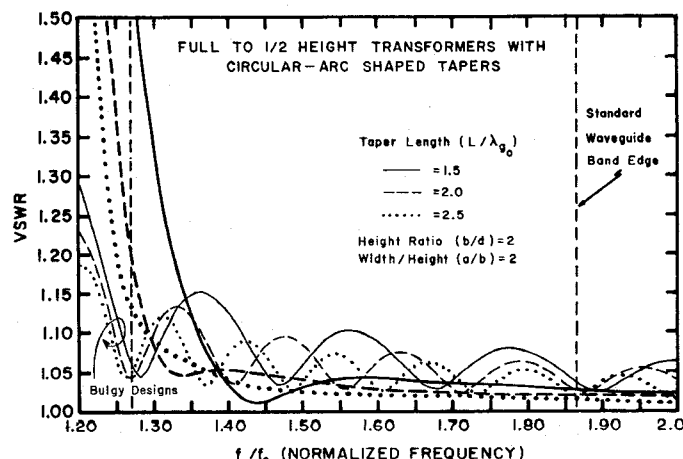
applied. Such an approach was used by Saad, Davies, and Davies [10] in the design of a Marie mode transformer.

To check the accuracy of the Johnson analysis using the Cohn [4] definition of characteristic impedance, the VSWR of each of the four transformers of Figs. 2 and 3 was computed and the results compared with the measured values. The agreement is fairly close except for very low values of VSWR. Fig. 4 shows the normalized TE-10 mode cutoff wavenumbers for the four transformers as functions of position along the length of the taper. The reasons for the dramatic rise in VSWR at the low frequency end of the waveguide band is now readily apparent as is the reason for the improvement in performance which results from the addition of a bulge to the narrow wall of the reduced height waveguide.

Figs. 5 and 6 contain some design curves for different length, full to one-quarter and one-half height transformers with circular-arc shaped tapers. Transitions with and without a bulge in the width are represented. In the case of the bulgy transformers, the bulge extends over the full length of the taper and increases the width of the reduced height waveguide by 25% at the maximum. The transformer lengths are normalized to the rectangular guide wavelength at the center of the waveguide band.

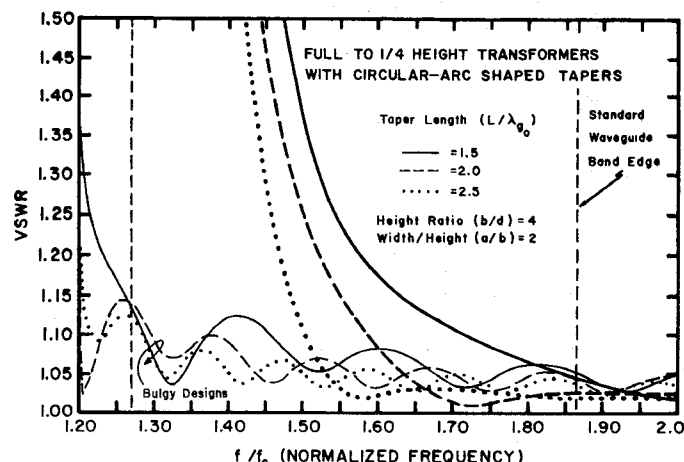
#### SUMMARY

A new type of easily fabricated H-plane waveguide transformer has been described. The transformer is particularly suitable for use at millimeter wavelengths where the fabrication of conventional stepped or tapered transitions is difficult and expensive. It can be formed in a split-block waveguide structure using a single setup on a milling machine. The block is split in the E-plane which has zero transverse current, hence poor contact along the joint line will cause no loss. The results of a theoretical analysis of the structure agree fairly well with measurements made on transformers at X-band. A complete analysis and full set of design curves appear in [11].



5. Predicted VSWR versus normalized frequency for six full to 1/2 height transformers with circular-arc shaped tapers. Three curves represent transformers with a bulge in the width of the reduced height waveguide. The transformer lengths are 1.5, 2, and 2.5 times the guide wavelength in rectangular waveguide at the center of the band and the frequency is normalized to the cutoff frequency of the rectangular waveguide. The slitting saw radius used to produce a particular taper is given by:  $R/a = 13.461(L/\lambda_{g0})^{2+0.5}$ . The full height waveguide width to height ratio ( $a/b$ ) of these transitions is 2:1, characteristic of most millimeter waveguides. The two vertical lines indicate the normal operating band.

The full to one-half height design with a circular-arc shaped taper has been used in two solid-state frequency multipliers with outputs in WR-5 and WR-8 waveguide, and the full to one-quarter height design has been used in mixers at 115 GHz. Fabrication time for the devices was reduced dramatically by employing the new transformer. The design is also useful as a transition between the channel (or crossed) waveguide structure of [1,6,7] and conventional rectangular waveguide.



6. Predicted VSWR versus normalized frequency for six full to 1/4 height transformers with circular-arc shaped tapers. Three curves represent transformers with a bulge in the width of the reduced height waveguide which extends over the full length of the taper and increases the waveguide width by 25% at the maximum. The same conditions apply as in Fig. 5.

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